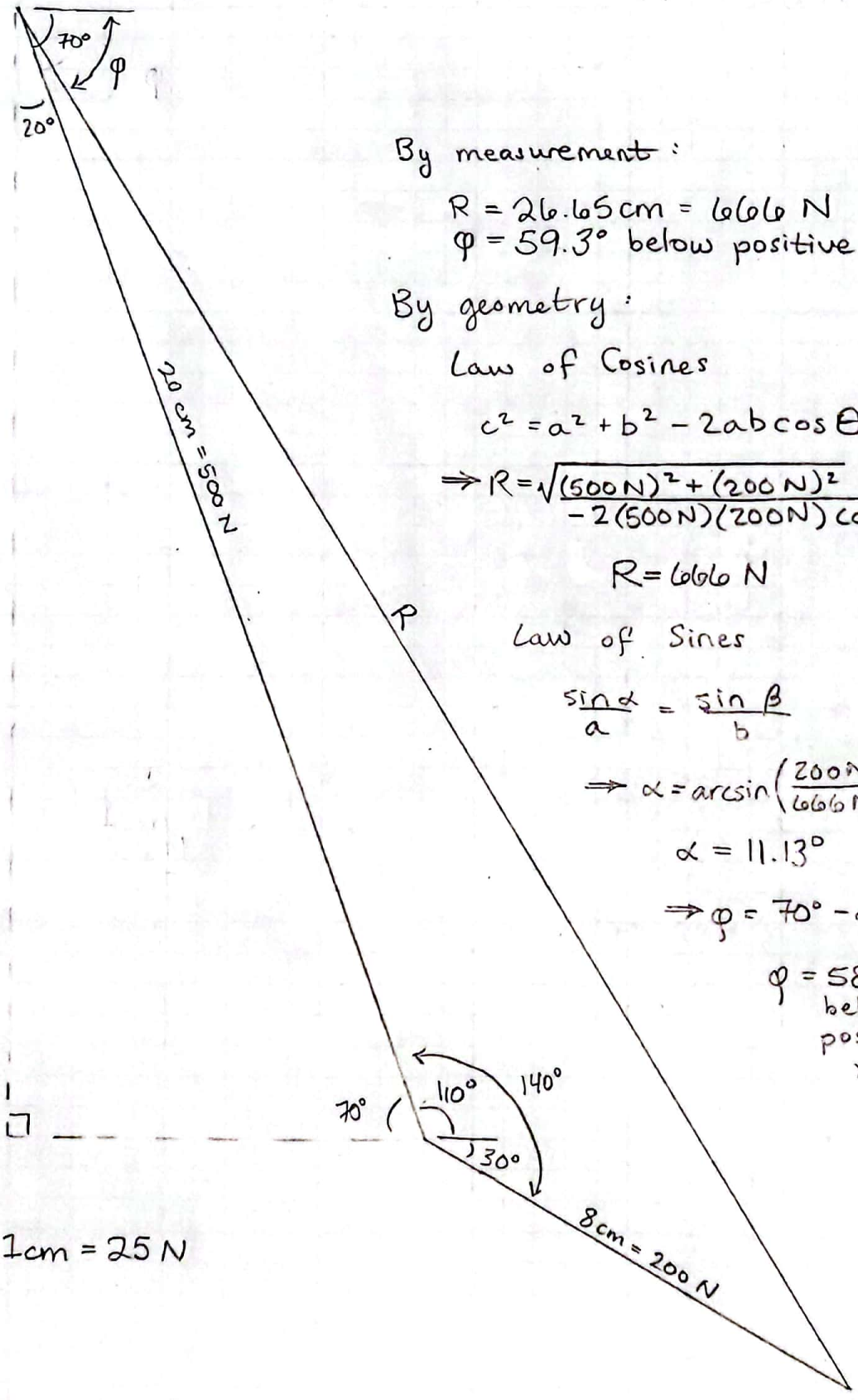
First Problem

Force Triangle:



By measurement:

$R = 26.65\,cm = 666\,N$

$\varphi = 59.3°$ below positive x-axis

By geometry:

Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

$$\Rightarrow R = \sqrt{(500N)^2 + (200N)^2 - 2(500N)(200N)\cos 140°}$$

$$R = 666\,N$$

Law of Sines

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b}$$

$$\Rightarrow \alpha = \arcsin\left(\frac{200N}{666N}\sin 140°\right)$$

$$\alpha = 11.13°$$

$$\Rightarrow \varphi = 70° - \alpha$$

$\varphi = 58.9°$ below positive x-axis

First Problem continued...

Divide into x-y components:

$F_1 = 500\,N$ @ 70° below +x-axis

$F_{1,x} = 500\,N \cos 70° = 171.0\,N$

$F_{1,y} = -500\,N \sin 70° = -469.8\,N$

$F_2 = 200\,N$ @ 30° below +x-axis

$F_{2,x} = 200\,N \cos 30° = 173.2\,N$

$F_{2,y} = 200\,N \sin 30° = -100\,N$

Vector sum for resultant:

$$\bar{R} = \bar{F_1} + \bar{F_2} = (171.0\,\hat{x} - 469.8\,\hat{y})\,N + (173.2\,\hat{x} - 100\,\hat{y})\,N$$
$$= (171.0 + 173.2)\,\hat{x}\,N - (469.8 + 100)\,\hat{y}\,N$$
$$= (344.2\,\hat{x} - 569.8\,\hat{y})\,N$$

magnitude

$$R = \sqrt{(344.2\,N)^2 + (-569.8\,N)^2} = 666\,N$$

direction

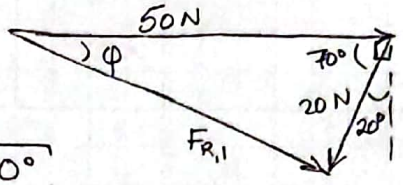
$$\varphi = \arctan \frac{569.8\,N}{344.2\,N} = 58.9° \text{ below } +x\text{-axis}$$

Second Problem

Solve graphically or mathematically. Here is a graphical/geometric solution.

Find $F_{R,1} = F_2 + F_3$:

NOT to scale.



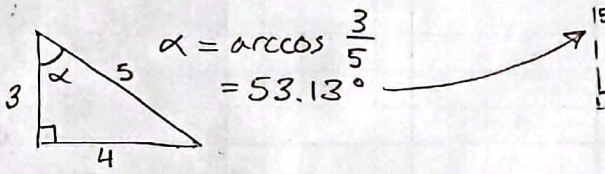
Law of Cosines

$$F_{R,1} = \sqrt{(50\,N)^2 + (20\,N)^2 - 2(50\,N)(20\,N)\cos 70°}$$

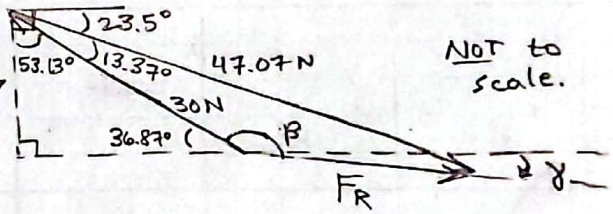
$$\Rightarrow F_{R,1} = 47.07\,N$$

Law of Sines $\quad \frac{\sin\varphi}{20\,N} = \frac{\sin 70°}{47.07\,N} \Rightarrow \varphi = 23.5°$

Find $F_R = F_{R,1} + F_1$:



$$\alpha = \arccos\frac{3}{5} = 53.13°$$



NOT to scale.

Law of Cosines

$$F_R = \sqrt{(47.07\,N)^2 + (30\,N)^2 + 2(47.07\,N)(30\,N)\cos 13.37°}$$

$$\Rightarrow F_R = 19.18\,N$$

$$\cos\beta = \frac{(30\,N)^2 + (19.18\,N)^2 - (47.07\,N)^2}{2(30\,N)(19.18\,N)} \Rightarrow \beta = 145.4°$$

$$\Rightarrow \gamma = \beta + 36.87° - 180°$$

$$= 145.4° + 36.87° - 180°$$

$$= 2.27° \text{ below } +x\text{-axis}$$