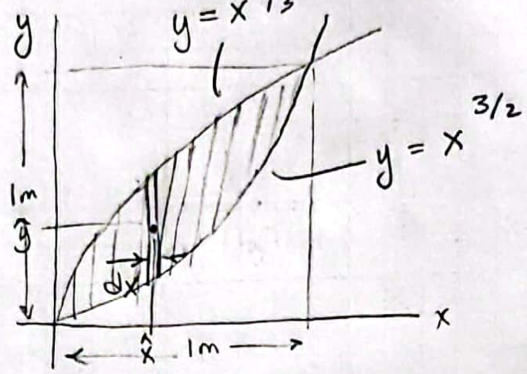
4/13/22 | SI Statics Worksheet #10 | Solution

First Problem



$$\hat{x} = x$$

$$\hat{y} = x^{3/2} + \frac{x^{2/3} - x^{3/2}}{2}$$

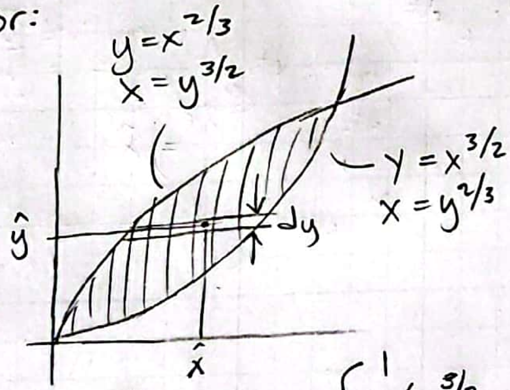
$$dA = (x^{2/3} - x^{3/2})\,dx$$

$$\bar{x} = \frac{\int \hat{x}\,dA}{\int dA} = \frac{\int_0^1 x(x^{2/3} - x^{3/2})\,dx}{\int_0^1 x^{2/3} - x^{3/2}\,dx} = \frac{\int_0^1 x^{5/3} - x^{5/2}\,dx}{\int_0^1 x^{2/3} - x^{3/2}\,dx}$$

$$= \frac{\frac{3}{8}x^{8/3}\Big|_0^1 - \frac{2}{7}x^{7/2}\Big|_0^1}{\frac{3}{5}x^{5/3}\Big|_0^1 - \frac{2}{5}x^{5/2}\Big|_0^1} = \frac{0.089286}{0.2} = \boxed{0.446}$$

alternate solution

or:



$$\hat{x} = y^{3/2} + \frac{y^{2/3} - y^{3/2}}{2}$$

$$\hat{y} = y$$

$$dA = (y^{2/3} - y^{3/2})\,dy$$

$$\bar{x} = \frac{\int \bar{x}\,dA}{\int dA} = \frac{\int_0^1 \left(y^{3/2} + \frac{y^{2/3} - y^{3/2}}{2}\right)(y^{2/3} - y^{3/2})\,dy}{\int_0^1 (y^{2/3} - y^{3/2})\,dy}$$

denominator: $\int_0^1 (y^{2/3} - y^{3/2})\,dy = 0.2$

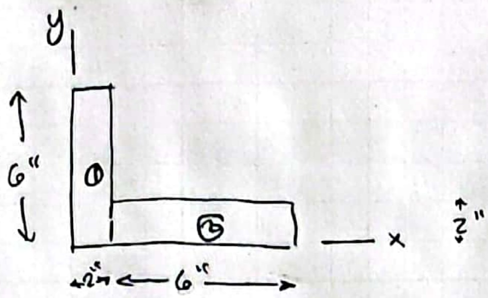
numerator: $\int_0^1 y^{13/6} + y^{2/3}\left(\frac{y^{2/3} - y^{3/2}}{2}\right) - y^3 - y^{3/2}\left(\frac{y^{2/3} - y^{3/2}}{2}\right) dy$

$$= \int_0^1 \cancel{y^{13/6}} + \frac{1}{2}y^{4/3} - \cancel{\frac{1}{2}y^{13/6}} - y^3 - \cancel{\frac{1}{2}y^{13/6}} + \frac{1}{2}y^3\,dy$$

$$= \int_0^1 \frac{1}{2}y^{4/3} - \frac{1}{2}y^3\,dy = \frac{1}{2}\cdot\frac{3}{7}y^{7/3}\Big|_0^1 - \frac{1}{2}\cdot\frac{1}{4}y^4\Big|_0^1 = 0.089286$$

$$\Rightarrow \bar{x} = \frac{0.089286}{0.2} = 0.446\,\text{m} \checkmark$$

Second Problem



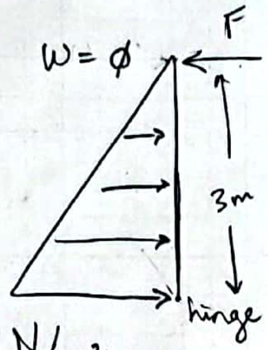
	$\bar{x}$	$\bar{y}$	A	$\bar{x}A$	$\bar{y}A$
①	1 in	3 in	12 in^2	12 in^3	36 in^2
②	5 in	1 in	12 in^2	60 in^3	12 in^3
			24 in^2	72 in^3	48 in^3

$$\bar{x} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{72 \text{ in}^3}{24 \text{ in}^2} = \boxed{3 \text{ in}}$$

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{48 \text{ in}^3}{24 \text{ in}^2} = \boxed{2 \text{ in}}$$

Third Problem

FBD:

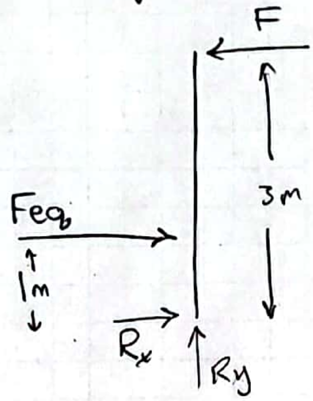


$w = \rho g z$
$= 47{,}088\ N/m^2$

$$F_{eq} = \frac{1}{2}(3m)(47088\ N/m^2)(1m)$$

$$= 70{,}632\ N$$

$\Downarrow$



$$\sum M_{hinge} = \emptyset$$

$$= (3m)F - (1m)F_{eq}$$

$$\Rightarrow \boxed{F = 24\ kN}$$