Week 7 5.1 Answers

1. $\sin^2 x - 4\sin x = 0$

$\sin x(\sin x - 4) = 0$ ∴ $\sin x = 0$ or $\sin x - 4 = 0$

$\sin x = 4$ sin never $= 0$

$\Rightarrow \sin x = 0$ at $\boxed{x = \pi k, k \in \mathbb{Z}}$

b. $2\cos^2 x - 5\cos x + 2 = 0$

$(2\cos x - 1)(\cos x - 2) = 0$

∴ $2\cos x - 1 = 0$ or $\cos x - 2 = 0$

$\cos x = \frac{1}{2}$ $\cos x = 2$ cos is never 2

$\Rightarrow \cos x = \frac{1}{2}$ at $\frac{\pi}{3} + \frac{5\pi}{3}$, so $\boxed{x = \frac{\pi}{3} + 2\pi k, k \in \mathbb{Z}}$ + $\boxed{x = \frac{5\pi}{3} + 2\pi k, k \in \mathbb{Z}}$

c. $2\sec^2\theta - 3\sec\theta - 2 = 0$

$(2\sec\theta + 1)(\sec\theta - 2)$

∴ $2\sec\theta + 1 = 0$ or $\sec\theta - 2 = 0$

$\sec\theta = -\frac{1}{2}$ ← Same as $\cos = -2$, doesn't exist $\sec\theta = 2$

$\Rightarrow \sec\theta = 2$ at $\frac{\pi}{3} + \frac{5\pi}{3}$, so $\theta = \boxed{\frac{\pi}{3} + 2\pi k, k \in \mathbb{Z}}$ or $\boxed{\theta = \frac{5\pi}{3} + 2\pi k, k \in \mathbb{Z}}$

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2. a. $-5\cos\theta = -2\sin^2\theta + 4$

$-5\cos\theta = -2(1 - \cos^2\theta) + 4$

$-5\cos\theta = -2 + 2\cos^2\theta + 4$

$2\cos^2\theta + 5\cos\theta + 2 = 0$

$2\cos\theta + 1 = 0$ or ~~$\cos\theta + 2 = 0$~~

$\cos\theta = -\frac{1}{2}$

∴ $\boxed{x = \frac{2\pi}{3} + 2\pi k, k \in \mathbb{Z}}$ or $\boxed{x = \frac{4\pi}{3} + 2\pi k, k \in \mathbb{Z}}$

b. $\tan^2 x + 2\sec x + 1 = 0$

$\sec^2 x + 2\sec x + 1 = 0$

$\sec x(\sec x + 2) = 0$

~~$\sec x = 0$~~ or $\sec x + 2 = 0$

$\sec x = -2$

$\Rightarrow \sec x = -2$ at $\frac{2\pi}{3} + \frac{4\pi}{3}$, so $\boxed{x = \frac{2\pi}{3} + 2k\pi, k \in \mathbb{Z}}$ or $\boxed{x = \frac{4\pi}{3} + 2\pi k, k \in \mathbb{Z}}$

c. $4\sin^2\theta = 5 + 4\cos\theta$

$4(1 - \cos^2\theta) = 5 + 4\cos\theta$

$4 - 4\cos^2\theta = 5 + 4\cos\theta$

$4 - 4\cos^2\theta - 4\cos\theta - 5 = 0$

$-1[-4\cos^2\theta - 4\cos\theta - 1 = 0]$ -1

$(2\cos\theta + 1)(2\cos\theta + 1) = 0$

∴ $\cos\theta = -\frac{1}{2}$

∴ $\boxed{\theta = \frac{2\pi}{3} + 2\pi k, k \in \mathbb{Z}}$ or $\boxed{\theta = \frac{4\pi}{3} + 2\pi k, k \in \mathbb{Z}}$

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3. a. $\sin^2 x - \cos^2 x = 1 - 2\cos^2 x$

$(1 - \cos^2 x) - \cos^2 x = 1 - 2\cos^2 x$

$1 - 2\cos^2 x = 1 - 2\cos^2 x$

pythagorean Identity

b. $\sin^2 x + 3\cos^2 x = 3 - 2\sin^2 x$

$\sin^2 x + 3(1 - \sin^2 x) = 3 - 2\sin^2 x$

$\sin^2 x + 3 - 3\sin^2 x = 3 - 2\sin^2 x$

$3 - 2\sin^2 x = 3 - 2\sin^2 x$

pythagorean Identity