Week 8 SI Answers

1. a. $\tan(\pi - x) = -\tan x$

$$\Rightarrow \frac{\sin(\pi - x)}{\cos(\pi - x)}$$ Quotient rule

$$\Rightarrow \frac{\sin\pi\cos x - \cos\pi\sin x}{\cos\pi\cos x + \sin\pi\sin x}$$ Sum + difference

$$\Rightarrow \frac{0\cdot\cos x + 1\cdot\sin x}{-1\cdot\cos x + 0\cdot\sin x}$$ evaluate

$$\Rightarrow \frac{\sin x}{-\cos x} = \boxed{-\tan x}$$ Quotient Rule

b. $\sin\left(\frac{\pi}{3} + x\right) - \sin\left(\frac{\pi}{3} - x\right) = \sin x$

$$\Rightarrow \cancel{\sin\frac{\pi}{3}\cos x} + \cos\frac{\pi}{3}\sin x - \left[\cancel{\sin\frac{\pi}{3}\cos x} - \cos\frac{\pi}{3}\sin x\right]$$ Sum + difference

$$\Rightarrow 2\cos\frac{\pi}{3}\sin x$$ Evaluate

$$\Rightarrow 2\cdot\frac{1}{2}\sin x = \boxed{\sin x}$$

c. $\dfrac{\cos(\pi + x)}{\cos\left(\frac{3\pi}{2} - x\right)} = \cot x$

$$\Rightarrow \frac{\cos\pi\cos x - \sin\pi\sin x}{\cos\frac{3\pi}{2}\cos x + \sin\frac{3\pi}{2}\sin x}$$ Sum + difference

$$\Rightarrow \frac{-1\cdot\cos x - 0\cdot\sin x}{0\cdot\cos x + -1\cdot\sin x}$$ evaluate

$$\Rightarrow \frac{-\cos x}{-\sin x} = \cot x$$ quotient rule

2.

a. $\cos x \cos 3x - \sin x \sin 3x = -1$

$\Rightarrow \cos x \cos 3x - \sin x \sin 3x = \cos(x+3x)$ Sum + difference

$\therefore \cos 4x = -1$

$\therefore 4x = \pi + 2\pi k,\ k \in Z$

$x = \frac{\pi}{4} + \frac{\pi}{2}k,\ k \in Z$, so on $[0, 2\pi)$. $\boxed{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

b. $2\sin 4x \cos 2x - 2\cos 4x \sin 2x = \sqrt{3}$

$\Rightarrow 2(\sin 4x \cos 2x - \cos 4x \sin 2x) = \sqrt{3}$

$\Rightarrow \sin 4x \cos 2x - \cos 4x \sin 2x = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin(2x) = \frac{\sqrt{3}}{2}$ Sum + difference

$\therefore x = \frac{\pi}{6} + \pi k,\ k \in Z$

$+$

$x = \frac{\pi}{3} + \pi k,\ k \in Z$

$\therefore \boxed{\frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}}$

3.

a. $1 - 2\cos^2\frac{x}{2} = \cos^2 x$

$\Rightarrow 1 - 2 \cdot \frac{1+\cos x}{2} = \cos^2 x$ half-angle formula

$\Rightarrow 1 - (1 + \cos x) = \cos^2 x$

$\Rightarrow \cos^2 x + \cos x = 0$

$\cos x(\cos x + 1) = 0$

$\therefore$ $\cos x = 0$ $\downarrow$ $\boxed{x = \frac{\pi}{2} + \pi k,\ k \in Z}$

or

$\cos x + 1 = 0$

$\cos x = -1$ $\downarrow$ $\boxed{x = \pi + 2\pi k,\ k \in Z}$

b. $\frac{2\cot^2 x - \csc^2 x}{1 + \cot^2} = \cos 2x$

$\Rightarrow \frac{2\cot^2 x - \csc^2 x}{\csc^2 x}$ pythagorean

$\Rightarrow \frac{2\cot^2 x}{\csc^2 x} - \frac{\csc^2 x}{\csc^2 x}$

$\Rightarrow \frac{2\cot^2 x}{\csc^2 x} - 1$

$\Rightarrow \frac{\frac{2\cos^2 x}{\sin^2 x}}{\frac{1}{\sin^2 x}} - 1$ ← quotient rule ← reciprocal identity

$\Rightarrow 2\cos^2 x - 1$ Algebra

$\Rightarrow \boxed{\cos 2x}$ Double-angle

c. $\dfrac{2\tan x}{1+\tan^2 x} = \sin 2x$

$\Rightarrow \dfrac{2\tan x}{\sec^2 x}$ pythagorean Identity

$\Rightarrow \dfrac{\frac{2\sin x}{\cos x}}{\sec^2 x}$ quotient rule

$\Rightarrow \dfrac{\frac{2\sin x}{\cos x}}{\frac{1}{\cos^2 x}}$ reciprocal Identity

$\Rightarrow \dfrac{2\sin x}{\cos x} \cdot \cos^2 x$ Algebra

$\Rightarrow 2\sin x\cos x$ Evaluate

$\Rightarrow \boxed{\sin 2x}$