FINAL EXAM REVIEW

1. a.

$r = .2m \Rightarrow 20\text{ cm}$

$\theta = ?$

$s = 19\text{cm}$

$\theta = \frac{s}{r} \Rightarrow \frac{19}{20} = \boxed{.95 \text{ or } 54.43^\circ}$

b.

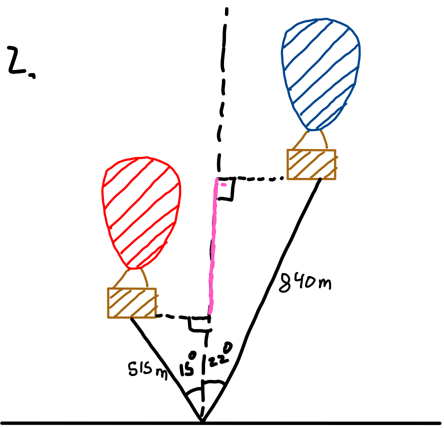
$r = 400\text{cm} \Rightarrow 40\text{m}$

$\theta = 96^\circ \Rightarrow 1.675...$

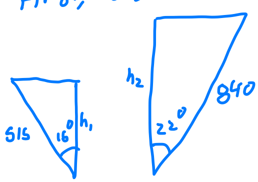
$s = ?$

$s = \theta r \Rightarrow s = 1.675(40) = \boxed{67.02\text{m}}$

2.



First, break up the 2 triangles



$\Rightarrow \cos(15^\circ) = \frac{h_1}{515}$

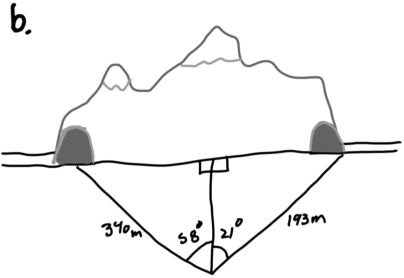
$\therefore 515\cos(15^\circ) = h_1$

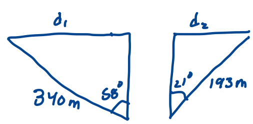
$\Rightarrow \cos(22^\circ) = \frac{h_2}{840}$

$\therefore 840\cos(22^\circ) = h_2$

$h_2 - h_1 \Rightarrow 840\cos(22^\circ) - 515\cos(16^\circ) = \boxed{281.382\text{m}}$

b.





$\sin 58^\circ = \frac{d_1}{340}$ $\quad$ $\sin 21^\circ = \frac{d_2}{193}$

$340\sin 58^\circ = d_1$ $\quad$ $193\sin 21^\circ = d_2$

$d_1 + d_2 \Rightarrow 340\sin 58^\circ + 193\sin 21^\circ = \boxed{357.501}$

3.

a. $\cos(-135^\circ) = \cos(225^\circ) = \boxed{-\frac{\sqrt{2}}{2}}$

* -135 is coterminal with 225

$\cos(2 \cdot 135^\circ) = \cos(270^\circ) = \boxed{0}$

$\cos^2(135) = (\cos(135^\circ))^2 = \left(\frac{\sqrt{2}}{2}\right)^2 = \boxed{\frac{1}{2}}$

b. $\sin\left(-\frac{\pi}{4}\right) = \sin\left(\frac{7\pi}{4}\right) = \boxed{-\frac{\sqrt{2}}{2}}$

$\sin^2 \frac{\pi}{4} = \left(\sin \frac{\pi}{4}\right)^2 = \left(\frac{\sqrt{2}}{2}\right)^2 = \boxed{\frac{1}{2}}$

$\sin 2 \cdot \frac{\pi}{4} = \sin \frac{\pi}{2} = \boxed{1}$

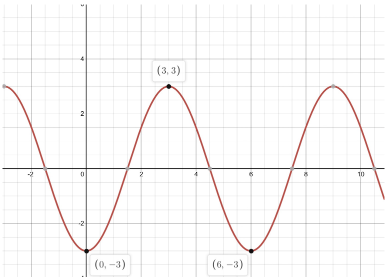
4.a. use the format $d = a\cos bt$ [No d value given]

- find a + b

=> amplitude given as 3cm, so $|a| = 3$
   - but, were given that it starts at -3cm, so
     $a = -3$

=> were given the period (6s), so $\frac{2\pi}{b} = 6 \Rightarrow b = \frac{2\pi}{6} = \frac{\pi}{3}$

$\therefore \boxed{d = -3\cos\left(\frac{\pi}{3}t\right)}$

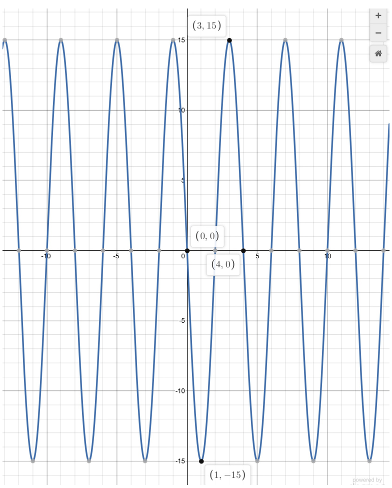


b. - again, find a + b in for $d = -15\sin\left(\frac{\pi}{2}t\right)$

=> a is given as 15, so $|a| = 15$
   - but, it says it initially moves downwards (negative),
     so a must be -15

=> $\frac{2\pi}{b} = 4 \therefore b = \frac{2\pi}{4} = \frac{\pi}{2}$

$\therefore \boxed{d = -15\sin\left(\frac{\pi}{2}t\right)}$



5. a. $\cot x + 1 = 0$ or $\csc x - 1 = 0$

$\cot x = -1$

$\cot$ is -1 at $\boxed{\frac{3\pi}{4} + \frac{7\pi}{4}}$

$\csc x = 1$

$\csc$ is 1 at $\boxed{\frac{\pi}{2}}$

b. $\cot x - \sqrt{3} = 0$ or $2\cos x - \sqrt{2} = 0$

$\cot x = \sqrt{3}$

$\cot$ is $\sqrt{3}$ at $\boxed{\frac{\pi}{6} + \frac{7\pi}{6}}$

$\cos x = \frac{\sqrt{2}}{2}$

$\cos x$ is $\frac{\sqrt{2}}{2}$ at $\boxed{\frac{\pi}{4} + \frac{7\pi}{4}}$

c. $2\sin x + \sqrt{2} = 0$ or $2\cos x - \sqrt{3} = 0$

$\sin x = -\frac{\sqrt{2}}{2}$

$\sin x = -\frac{\sqrt{2}}{2}$ at $\boxed{\frac{5\pi}{4} + \frac{7\pi}{4}}$

$\cos x = \frac{\sqrt{3}}{2}$

$\cos x = \frac{\sqrt{3}}{2}$ at $\boxed{\frac{\pi}{6} + \frac{11\pi}{6}}$

6. a. $3\cot^2 x - 1 = 0$ $\therefore$ $\cot = \pm\frac{1}{\sqrt{3}}$ or $\pm\frac{\sqrt{3}}{3}$

$\cot$ is $\frac{\sqrt{3}}{3}$ at $\boxed{\frac{\pi}{3} + \frac{4\pi}{3}}$, + $-\frac{\sqrt{3}}{3}$ at $\boxed{\frac{2\pi}{3} + \frac{5\pi}{3}}$

b. $\sec^2 x - 2 = 0$ $\therefore$, $\sec x = \pm\sqrt{2}$

$\sec x$ is $\sqrt{2}$ at $\boxed{\frac{\pi}{4} + \frac{7\pi}{4}}$ + $-\sqrt{2}$ at $\boxed{\frac{3\pi}{4} + \frac{5\pi}{4}}$

c. $4\cos^2 x - 3 = 0$ $\therefore$ $\cos = \pm\sqrt{\frac{3}{4}}$ or $\pm\frac{\sqrt{3}}{2}$

$\cos x = \frac{\sqrt{3}}{2}$ at $\boxed{\frac{\pi}{6} + \frac{11\pi}{6}}$, + $-\frac{\sqrt{3}}{2}$ at $\boxed{\frac{5\pi}{6} + \frac{7\pi}{6}}$

7. a. $\tan(\pi - x) = -\tan x$

$$\Rightarrow \frac{\sin(\pi - x)}{\cos(\pi - x)}$$ Quotient rule

$$\Rightarrow \frac{\sin\pi\cos x - \cos\pi\sin x}{\cos\pi\cos x + \sin\pi\sin x}$$ Sum + difference

$$\Rightarrow \frac{0\cdot\cos x + 1\cdot\sin x}{-1\cdot\cos x + 0\cdot\sin x}$$ evaluate

$$\Rightarrow \frac{\sin x}{-\cos x} = \boxed{-\tan x}$$ Quotient Rule

b. $\sin\left(\frac{\pi}{3} + x\right) - \sin\left(\frac{\pi}{3} - x\right) = \sin x$

$$\Rightarrow \cancel{\sin\frac{\pi}{3}\cos x} + \cos\frac{\pi}{3}\sin x - \left[\cancel{\sin\frac{\pi}{3}\cos x} - \cos\frac{\pi}{3}\sin x\right]$$ Sum + difference

$$\Rightarrow 2\cos\frac{\pi}{3}\sin x$$ Evaluate

$$\Rightarrow 2\cdot\frac{1}{2}\sin x = \boxed{\sin x}$$

c. $\dfrac{\cos(\pi + x)}{\cos\left(\frac{3\pi}{2} - x\right)} = \cot x$

$$\Rightarrow \frac{\cos\pi\cos x - \sin\pi\sin x}{\cos\frac{3\pi}{2}\cos x + \sin\frac{3\pi}{2}\sin x}$$ Sum + difference

$$\Rightarrow \frac{-1\cdot\cos x - 0\cdot\sin x}{0\cdot\cos x + -1\cdot\sin x}$$ evaluate

$$\Rightarrow \frac{-\cos x}{-\sin x} = \cot x$$ quotient rule

8. a.

$A = 115^0$ $\quad a = ?$

$\boxed{B = 30^0}$ $\quad b = ?$

$C = 35^0$ $\quad c = 70$

$$\frac{a}{\sin(115)} = \frac{70}{\sin(35)} \Rightarrow a = \frac{70}{\sin(35)} \cdot \sin(115)$$

$$\boxed{a = 110.6}$$

$$\frac{b}{\sin(30)} = \frac{70}{\sin(35)} \Rightarrow b = \frac{70}{\sin(35)} \cdot \sin(30)$$

$$\boxed{b = 61}$$

b.

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$b = \sqrt{a^2+c^2-2ac\cos B}$$

$$b = \sqrt{16^2+61^2-2(16)(61)\cos(117)}$$

$$b = \boxed{64.74}$$

Next, find angle A or C

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$$

$$\sin(A) = \frac{\sin(B)\, a}{b}$$

$$A = \sin^{-1}\left(\frac{16\sin(117)}{69.74}\right) = \boxed{11.796^0}$$

$$A + B + C = 180^0$$

$$11.796 + 117^0 + C = 180$$

$$\boxed{C = 51.204}$$

9.

a. $14 = 7.7 + 10.7\sin\left(\frac{2\pi}{63}t\right)$

$$\frac{6.3}{10.7} = \sin\left(\frac{2\pi}{63}t\right)$$ - let $u = \frac{2\pi}{63}t$

$$\sin(u) = \frac{6.3}{10.7} \Rightarrow u = \sin^{-1}\left(\frac{6.3}{10.7}\right)$$

$$\frac{2\pi}{63}t = \sin^{-1}\left(\frac{6.3}{10.7}\right) \quad + \quad \frac{2\pi}{63}t = \pi - \sin^{-1}\left(\frac{6.3}{10.7}\right)$$

$$t = \frac{63}{2\pi}\sin^{-1}\left(\frac{6.3}{10.7}\right) + \quad t = \frac{63}{2\pi}\left(\pi - \sin^{-1}\left(\frac{6.3}{10.7}\right)\right)$$

$\boxed{t = 6 \text{ days}}$ $\quad \boxed{t = 25 \text{ days}}$

- let $u = 2\theta$

b. $R(\theta) = \frac{V_0^2\sin 2\theta}{32} \Rightarrow 435 = \frac{160^2\sin 2\theta}{32} \Rightarrow \sin(2\theta) = \frac{435}{800}$

$$\sin(u) = \frac{435}{800} \Rightarrow u = \sin^{-1}\left(\frac{435}{800}\right)$$

$\therefore$

$$2\theta = \sin^{-1}\left(\frac{435}{800}\right) \quad + \quad 2\theta = \pi - \sin^{-1}\left(\frac{435}{800}\right)$$

$$\theta = \frac{\sin^{-1}\left(\frac{435}{800}\right)}{2} \qquad \theta = \frac{\pi - \sin^{-1}\left(\frac{435}{800}\right)}{2}$$

$\boxed{\theta = 1.28}$

$\boxed{\theta = .29}$